Beautiful Arithmetic

The Number Symphonies of Royal Heath: Symphony no. 1

$$1 \times 1 = 1$$
$$11 \times 11 = 121$$
$$111 \times 111 = 12321$$
$$1111 \times 1111 = 1234321$$
$$11111 \times 11111 = 123454321$$
$$111111 \times 111111 = 12345654321$$
$$1111111 \times 1111111 = 1234567654321$$
$$11111111 \times 11111111 = 123456787654321$$
$$111111111 \times 111111111 = 12345678987654321$$

$$1 = \frac{1 \times 1}{1} = \frac{1^2}{1^2}$$
$$121 = \frac{22 \times 22}{1+2+1} = \frac{22^2}{2^2}$$
$$12321 = \frac{333 \times 333}{1+2+3+2+1} = \frac{333^2}{3^2}$$
$$1234321 = \frac{4444 \times 4444}{1+2+3+4+3+2+1} = \frac{4444^2}{4^2}$$
$$123454321 = \frac{55555 \times 55555}{1+2+3+4+5+4+3+2+1} = \frac{55555^2}{5^2}$$
$$12345654321 = \frac{666666 \times 666666}{1+2+3+4+5+6+5+4+3+2+1} = \frac{666666^2}{6^2}$$
$$1234567654321 = \frac{7777777 \times 7777777}{1+2+3+4+5+6+7+6+5+4+3+2+1} = \frac{7777777^2}{7^2}$$
$$123456787654321 = \frac{88888888 \times 88888888}{1+2+3+4+5+6+7+8+7+6+5+4+3+2+1} = \frac{88888888^2}{8^2}$$
$$12345678987654321 = \frac{999999999 \times 999999999}{1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1} = \frac{999999999^2}{9^2}$$

The Number Symphonies of Royal Heath: Symphony no. 2

$$\begin{aligned}
1 + 1 + 1 &= 3 \\
2 + 2 + 2 &= 6 \\
3 + 3 + 3 &= 9 \\
4 + 4 + 4 &= 12 \\
5 + 5 + 5 &= 15 \\
6 + 6 + 6 &= 18 \\
7 + 7 + 7 &= 21 \\
8 + 8 + 8 &= 24 \\
9 + 9 + 9 &= 27
\end{aligned}$$

$$\begin{aligned}
3 \times 37 &= 111 \\
6 \times 37 &= 222 \\
9 \times 37 &= 333 \\
12 \times 37 &= 444 \\
15 \times 37 &= 555 \\
18 \times 37 &= 666 \\
21 \times 37 &= 777 \\
24 \times 37 &= 888 \\
27 \times 37 &= 999
\end{aligned}$$

$$\begin{aligned}
1 \times 9 + 2 &= 11 \\
12 \times 9 + 3 &= 111 \\
123 \times 9 + 4 &= 1111 \\
1234 \times 9 + 5 &= 11111 \\
12345 \times 9 + 6 &= 111111 \\
123456 \times 9 + 7 &= 1111111 \\
1234567 \times 9 + 8 &= 11111111 \\
12345678 \times 9 + 9 &= 111111111 \\
123456789 \times 9 + 10 &= 1111111111
\end{aligned}$$

$$\begin{aligned}
1 \times 8 + 1 &= 9 \\
12 \times 8 + 2 &= 98 \\
123 \times 8 + 3 &= 987 \\
1234 \times 8 + 4 &= 9876 \\
12345 \times 8 + 5 &= 98765 \\
123456 \times 8 + 6 &= 987654 \\
1234567 \times 8 + 7 &= 9876543 \\
12345678 \times 8 + 8 &= 98765432 \\
123456789 \times 8 + 9 &= 987654321
\end{aligned}$$

$$\begin{aligned}
9 \times 9 + 7 &= 88 \\
9 \times 98 + 6 &= 888 \\
9 \times 987 + 5 &= 8888 \\
9 \times 9876 + 4 &= 88888 \\
9 \times 98765 + 3 &= 888888 \\
9 \times 987654 + 2 &= 8888888 \\
9 \times 9876543 + 1 &= 88888888 \\
9 \times 98765432 \ \ 0 &= 888888888 \\
9 \times 987654321 - 1 &= 8888888888
\end{aligned}$$

The Number Symphonies of Royal Heath: Symphony no. 3

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$
$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$
$$36 + 37 + 38 + 39 + 40 + 41 + 42 = 43 + 44 + 45 + 46 + 47 + 48$$
$$49 + 50 + 51 + 52 + 53 + 54 + 55 + 56 = 57 + 58 + 59 + 60 + 61 + 62 + 63$$
$$64 + 65 + 66 + 67 + 68 + 69 + 70 + 71 + 72 = 73 + 74 + 75 + 76 + 77 + 78 + 79 + 80$$
$$81 + 82 + 83 + 84 + 85 + 86 + 87 + 88 + 89 + 90 = 91 + 92 + 93 + 94 + 95 + 96 + 97 + 98 + 99$$

1. In any line, the number of terms to the left of the equal sign is one more than the number of terms to the right of the equal sign. The product of these two numbers is the middle term of the line.
2. The difference between any term and the one immediately below it is always equal between any two lines, and the difference increases by 2 from line to line.
3. The first term of every line is a perfect square.

The Number Symphonies of Royal Heath: Symphony no. 4

$$3^2 + 4^2 = 5^2$$
$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$
$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2$$
$$36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$$
$$55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$$
$$78^2 + 79^2 + 80^2 + 81^2 + 82^2 + 83^2 + 84^2 = 85^2 + 86^2 + 87^2 + 88^2 + 89^2 + 90^2$$

1. In any line, the number of terms to the left of the equal sign is one more than the number of terms to the right of the equal sign. The product of these two numbers is *half* the middle term of the line.
2. From the last term of a line to the first term of the next line, there is, instead of an increase by 1, an increase by an odd integer which itself increases by 2 from line to line.

The Number Symphonies of Royal Heath: Symphony no. 5

$19 \times 1 = 19$	$1 + 9 = 10$	$1 + 0 = 1$	$1089 \times 1 = 1089$	$1 + 0 + 8 + 9 = 18$
$19 \times 2 = 38$	$3 + 8 = 11$	$1 + 1 = 2$	$1089 \times 2 = 2178$	$2 + 1 + 7 + 8 = 18$
$19 \times 3 = 57$	$5 + 7 = 12$	$1 + 2 = 3$	$1089 \times 3 = 3267$	$3 + 2 + 6 + 7 = 18$
$19 \times 4 = 76$	$7 + 6 = 13$	$1 + 3 = 4$	$1089 \times 4 = 4356$	$4 + 3 + 5 + 6 = 18$
$19 \times 5 = 95$	$9 + 5 = 14$	$1 + 4 = 5$	$1089 \times 5 = 5445$	$5 + 4 + 4 + 5 = 18$
$19 \times 6 = 114$	$11 + 4 = 15$	$1 + 5 = 6$	$1089 \times 6 = 6534$	$6 + 5 + 3 + 4 = 18$
$19 \times 7 = 133$	$13 + 3 = 16$	$1 + 6 = 7$	$1089 \times 7 = 7623$	$7 + 6 + 2 + 3 = 18$
$19 \times 8 = 152$	$15 + 2 = 17$	$1 + 7 = 8$	$1089 \times 8 = 8712$	$8 + 7 + 1 + 2 = 18$
$19 \times 9 = 171$	$17 + 1 = 18$	$1 + 8 = 9$	$1089 \times 9 = 9801$	$9 + 8 + 0 + 1 = 18$
$19 \times 10 = 190$	$19 + 0 = 19$	$1 + 9 = 10$	$1089 \times 10 = 10890$	$1 + 0 + 8 + 9 + 0 = 18$

The Number Symphonies of Royal Heath: Symphony no. 6

$$\frac{1}{7} = .\overline{142857}$$

$142857 \times 1 = 142857$	$142857 \times 7 = 999999$	$999999 \div 9 = 111111$
$142857 \times 3 = 428571$	$428571 \times 7 = 2999997$	$2999997 \div 9 = 333333$
$142857 \times 2 = 285714$	$285714 \times 7 = 1999998$	$1999998 \div 9 = 222222$
$142857 \times 6 = 857142$	$857142 \times 7 = 5999994$	$5999994 \div 9 = 666666$
$142857 \times 4 = 571428$	$571428 \times 7 = 3999996$	$3999996 \div 9 = 444444$
$142857 \times 5 = 714285$	$714285 \times 7 = 4999995$	$4999995 \div 9 = 555555$

The Number Symphonies of Royal Heath: Symphony no. 7

$$\frac{1}{13} = .\overline{076923}$$

$76923 \times 1 = 76923$	$76923 \times 13 = 999999$	$999999 \div 9 = 111111$
$76923 \times 10 = 769230$	$769230 \times 13 = 9999990$	$9999990 \div 9 = 1111110$
$76923 \times 9 = 692307$	$692307 \times 13 = 8999991$	$8999991 \div 9 = 999999$
$76923 \times 12 = 923076$	$923076 \times 13 = 11999988$	$11999988 \div 9 = 1333332$
$76923 \times 3 = 230769$	$230769 \times 13 = 2999997$	$2999997 \div 9 = 333333$
$76923 \times 4 = 307692$	$307692 \times 13 = 3999996$	$3999996 \div 9 = 444444$
$76923 \times 2 = 153846$	$153846 \times 13 = 1999998$	$1999998 \div 9 = 222222$
$76923 \times 7 = 538461$	$538461 \times 13 = 6999993$	$6999993 \div 9 = 777777$
$76923 \times 5 = 384615$	$384651 \times 13 = 4999995$	$4999995 \div 9 = 555555$
$76923 \times 11 = 846153$	$846153 \times 13 = 10999989$	$10999989 \div 9 = 1222221$
$76923 \times 6 = 461538$	$461538 \times 13 = 5999994$	$5999994 \div 9 = 666666$
$76923 \times 8 = 615384$	$615384 \times 13 = 7999992$	$7999992 \div 9 = 888888$

Patterns using all the digits

123456789 × 1 = 123456789	987654321 × 1 = 987654321
123456789 × 2 = 246913578	987654321 × 2 = 1975308642
123456789 × 3 = 370370367	987654321 × 3 = 2962962963
123456789 × 4 = 493827156	987654321 × 4 = 3950617284
123456789 × 5 = 617283945	987654321 × 5 = 4938271605
123456789 × 6 = 740740734	987654321 × 6 = 5925925926
123456789 × 7 = 864197523	987654321 × 7 = 6913580247
123456789 × 8 = 987654312	987654321 × 8 = 7901234568
123456789 × 9 = 1111111101	987654321 × 9 = 8888888889
123456789 × 10 = 1234567890	987654321 × 10 = 9876543210

12345679 × 1 = 12345679 (no 8)	12345679 × 9 = 111111111
12345679 × 2 = 24691358 (no 7)	24691358 × 9 = 222222222
12345679 × 3 = 37037037	37037037 × 9 = 333333333
12345679 × 4 = 49382716 (no 5)	49382716 × 9 = 444444444
12345679 × 5 = 61728395 (no 4)	61728395 × 9 = 555555555
12345679 × 6 = 74074074	74074074 × 9 = 666666666
12345679 × 7 = 86419753 (no 2)	86419753 × 9 = 777777777
12345679 × 8 = 98765432 (no 1)	98765432 × 9 = 888888888
12345679 × 9 = 111111111	111111111 × 9 = 999999999

157 × 28 = 4396

198 × 27 = 5346

297 × 18 = 5346

138 × 42 = 5796

483 × 12 = 5796

1738 × 4 = 6952

186 × 39 = 7254

1963 × 4 = 7852

$$2 = \frac{13458}{6729}$$
$$3 = \frac{17469}{5823}$$
$$4 = \frac{15768}{3942}$$
$$5 = \frac{13485}{2697}$$
$$6 = \frac{17658}{2943}$$
$$7 = \frac{16758}{2394}$$
$$8 = \frac{25496}{3187}$$
$$9 = \frac{57429}{6381}$$

$$9 = \frac{97524}{10836} = \frac{95823}{10647} = \frac{95742}{10638} = \frac{75249}{08361} = \frac{58239}{06471} = \frac{57429}{06381}$$

Kordemsky's infinite rectangular array no. 1

1	3	5	7	9	11	13	15	17	19
1	4	7	10	13	16	19	22	25	28
1	5	9	13	17	21	25	29	33	37
1	6	11	16	21	26	31	36	41	46
1	7	13	19	25	31	37	43	49	55
1	8	15	22	29	36	43	50	57	64
1	9	17	25	33	41	49	57	65	73
1	10	19	28	37	46	55	64	73	82
1	11	21	31	41	51	61	71	81	91
1	12	23	34	45	56	67	78	89	100

For reference, we imagine each row being numbered 1, 2, 3, etc. from the top, and each column being numbered 1, 2, 3, etc. from the left. The right-angled corridors, called *gnomons*, we also imagine as being numbered 1, 2, 3, etc. from the upper left corner.

1. In a given row, the numbers are increasing by a fixed amount, which is one more than the row number. The same is true of the columns, except that the amound of increase is one less than the column number.

2. A number on the central diagonal is the square of its row or column number.

3. The sum of the numbers in a gnomon is the cube of its gnomon number.

4. If we have any square, one of whose diagonals is part of the central diagonal, then the sum of the numbers in that square is a perfect square, and the number that it is the square of is the sum of the row or column numbers which pass through the square.

Kordemsky's infinite rectangular array no. 2

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

1. This is an ordinary multiplication table—that is, the number in a given spot is the product of its row number and its column number.

2. The number of each gnomon is the number on the outside of the gnomon, either at the left or on the top.

3. The sum of the numbers in a gnomon is the cube of the gnomon number.

4. If we have any square which has one corner at the upper left corner of the array, then the sum of the numbers in that square is a perfect square, and the number that it is the square of is the sum of the row or column numbers which pass through the square.

5. The last two statements imply the following:

$$(1)^2 = 1^3$$
$$(1+2)^2 = 1^3 + 2^3$$
$$(1+2+3)^2 = 1^3 + 2^3 + 3^3$$
$$(1+2+3+4)^2 = 1^3 + 2^3 + 3^3 + 4^3$$
$$(1+2+3+4+5)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$$
$$(1+2+3+4+5+6)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$$
$$(1+2+3+4+5+6+7)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$$
$$(1+2+3+4+5+6+7+8)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3$$
$$(1+2+3+4+5+6+7+8+9)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$$
$$(1+2+3+4+5+6+7+8+9+10)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$$

Kordemsky's infinite triangular array

							1										
						2	3	4									
					5	6	7	8	9								
				10	11	12	13	14	15	16							
			17	18	19	20	21	22	23	24	25						
		26	27	28	29	30	31	32	33	34	35	36					
	37	38	39	40	41	42	43	44	45	46	47	48	49				
50	51	52	53	54	55	56	57	58	59	60	61	62	63	64			
65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	
82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

For reference, we imagine the rows being numbered 1, 2, 3, etc. from the top.

1. The square of each row number is the last number in the row.
2. If the number n is immediately over m, then $n \times m$ is n rows directly below n.
3. If r is the row number, and c is the number in the exact center of the row, then $c = r^2 - r + 1$, and $c(r+1) = r^3 + 1$.

Digit properties

$37 + (3 \times 7) = 3^2 + 7^2$
$37 \times (3 + 7) = 3^3 + 7^3$
$48 \times (4 + 8) = 4^3 + 8^3$

$111 \times (11 + 1) = 11^3 + 1^3$
$147 \times (14 + 7) = 14^3 + 7^3$
$148 \times (14 + 8) = 14^3 + 8^3$

$37 \times 1 = 037$	$37 \times 2 = 074$	$37 \times 4 = 148$	$37 \times 5 = 185$	$37 \times 7 = 259$	$37 \times 8 = 296$
$37 \times 10 = 370$	$37 \times 11 = 407$	$37 \times 13 = 481$	$37 \times 14 = 518$	$37 \times 16 = 592$	$37 \times 17 = 629$
$37 \times 19 = 703$	$37 \times 20 = 740$	$37 \times 22 = 814$	$37 \times 23 = 851$	$37 \times 25 = 925$	$37 \times 26 = 962$

$9 + 9 = 18$	$24 + 3 = 27$	$47 + 2 = 49$	$497 + 2 = 499$
$9 \times 9 = 81$	$24 \times 3 = 72$	$47 \times 2 = 94$	$497 \times 2 = 994$

$12 \times 42 = 21 \times 24$
$12 \times 63 = 21 \times 36$
$12 \times 84 = 21 \times 48$
$13 \times 62 = 31 \times 26$
$13 \times 93 = 31 \times 39$
$14 \times 82 = 41 \times 28$
$23 \times 64 = 32 \times 46$

$23 \times 96 = 32 \times 69$
$24 \times 63 = 42 \times 36$
$24 \times 84 = 42 \times 48$
$26 \times 93 = 62 \times 39$
$34 \times 86 = 43 \times 68$
$36 \times 84 = 63 \times 48$
$46 \times 96 = 64 \times 69$

$$234256 = (2 + 3 + 4 + 2 + 5 + 6)^4$$

$$23 + 42 + 56 = 11^2 = 65 + 24 + 32$$

$09 = 3^2$
$1089 = 33^2$
$110889 = 333^2$
$111088889 = 3333^2$
$11110888889 = 33333^2$

$49 = 7^2$
$4489 = 67^2$
$444889 = 667^2$
$44448889 = 6667^2$
$4444488889 = 66667^2$

$16 = 4^2$
$1156 = 34^2$
$111556 = 334^2$
$11115556 = 3334^2$
$1111155556 = 33334^2$

$81 = 9^2$
$9801 = 99^2$
$998001 = 999^2$
$99980001 = 9999^2$
$9999800001 = 99999^2$

Power properties

$$2^1 + 3^1 + 7^1 = 1^1 + 5^1 + 6^1$$
$$2^2 + 3^2 + 7^2 = 1^2 + 5^2 + 6^2$$

$$0^1 + 5^1 + 5^1 + 10^1 = 1^1 + 2^1 + 8^1 + 9^1$$
$$0^2 + 5^2 + 5^2 + 10^2 = 1^2 + 3^2 + 8^2 + 9^2$$
$$0^3 + 5^3 + 5^3 + 10^3 = 1^3 + 3^3 + 8^3 + 9^3$$

$$1^1 + 4^1 + 12^1 + 13^1 + 20^1 = 2^1 + 3^1 + 10^1 + 16^1 + 19^1$$
$$1^2 + 4^2 + 12^2 + 13^2 + 20^2 = 2^2 + 3^2 + 10^2 + 16^2 + 19^2$$
$$1^3 + 4^3 + 12^3 + 13^3 + 20^3 = 2^3 + 3^3 + 10^3 + 16^3 + 19^3$$

$$1^1 + 6^1 + 7^1 + 17^1 + 18^1 + 23^1 = 2^1 + 3^1 + 11^1 + 13^1 + 21^1 + 22^1$$
$$1^2 + 6^2 + 7^2 + 17^2 + 18^2 + 23^2 = 2^2 + 3^2 + 11^2 + 13^2 + 21^2 + 22^2$$
$$1^3 + 6^3 + 7^3 + 17^3 + 18^3 + 23^3 = 2^3 + 3^3 + 11^3 + 13^3 + 21^3 + 22^3$$
$$1^4 + 6^4 + 7^4 + 17^4 + 18^4 + 23^4 = 2^4 + 3^4 + 11^4 + 13^4 + 21^4 + 22^4$$
$$1^5 + 6^5 + 7^5 + 17^5 + 18^5 + 23^5 = 2^5 + 3^5 + 11^5 + 13^5 + 21^5 + 22^5$$

$$4^2 + 5^2 + 6^2 = 2^2 + 3^2 + 8^2$$

$$42^2 + 53^2 + 68^2 = 24^2 + 35^2 + 86^2$$
$$42^2 + 58^2 + 63^2 = 24^2 + 85^2 + 36^2$$
$$43^2 + 52^2 + 68^2 = 34^2 + 25^2 + 86^2$$
$$43^2 + 58^2 + 62^2 = 34^2 + 85^2 + 26z2$$
$$48^2 + 52^2 + 63^2 = 84^2 + 25^2 + 36^2$$
$$48^2 + 53^2 + 62^2 = 84^2 + 35^2 + 26^2$$

$$13^1 + 42^1 + 53^1 + 57^1 + 68^1 + 97^1 = 79^1 + 86^1 + 75^1 + 35^1 + 24^1 + 31^1$$
$$13^2 + 42^2 + 53^2 + 57^2 + 68^2 + 97^2 = 79^2 + 86^2 + 75^2 + 35^2 + 24^2 + 31^2$$
$$13^3 + 42^3 + 53^3 + 57^3 + 68^3 + 97^3 = 79^3 + 86^3 + 75^3 + 35^3 + 24^3 + 31^3$$

Operations with five twos and four fours

$$1 = 2 + 2 - 2 - \frac{2}{2}$$
$$2 = 2 + 2 + 2 - 2 - 2$$
$$3 = 2 + 2 - 2 + \frac{2}{2}$$
$$4 = 2 \times 2 \times 2 - 2 - 2$$
$$5 = 2 + 2 + 2 - \frac{2}{2}$$
$$6 = 2 + 2 + 2 + 2 - 2$$
$$7 = \frac{22}{2} - 2 - 2$$
$$8 = 2 \times 2 \times 2 + 2 - 2$$
$$9 = 2 \times 2 \times 2 + \frac{2}{2}$$
$$10 = 2 + 2 + 2 + 2 + 2$$
$$11 = \frac{22}{2} + 2 - 2$$
$$12 = 2 \times 2 \times 2 + 2 + 2$$
$$13 = \frac{22 + 2 + 2}{2}$$
$$14 = 2 \times 2 \times 2 \times 2 - 2$$
$$15 = \frac{22}{2} + 2 + 2$$
$$16 = (2 \times 2 + 2 + 2) \times 2$$
$$17 = (2 \times 2)^2 + \frac{2}{2}$$
$$18 = 2 \times 2 \times 2 \times 2 \times +2$$
$$19 = 22 - 2 - \frac{2}{2}$$
$$20 = 22 + 2 - 2 - 2$$
$$21 = 22 - 2 + \frac{2}{2}$$
$$22 = 22 \times 2 - 22$$
$$23 = 22 + 2 - \frac{2}{2}$$
$$24 = 22 + 2 + 2 - 2$$
$$25 = 22 + 2 + \frac{2}{2}$$
$$26 = 2 \times \left(\frac{22}{2} + 2\right)$$

$$1 = \frac{4}{4} \times \frac{4}{4}$$
$$2 = \frac{4}{4} + \frac{4}{4}$$
$$3 = \frac{4 + 4 + 4}{4}$$
$$4 = 4 + (4 - 4) \times 4$$
$$5 = \frac{4 \times 4 + 4}{4}$$
$$6 = 4 + \frac{4 + 4}{4}$$
$$7 = 4 + 4 - \frac{4}{4}$$
$$8 = 4 + 4 + 4 - 4$$
$$9 = 4 + 4 + \frac{4}{4}$$
$$10 = \frac{44 - 4}{4}$$